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ABSTRACT
In the context of detecting changes in structural systems, multiple vibration-based damage detection methods have been proposed and successfully applied to both mechanical and civil structures over the past years. One of the popular schemes is based on a robust subspace-based residual and enjoys favorable statistical and computational properties, like invariance to changes in the excitation covariance and numerical stability. This paper presents an alternative Gaussian residual that is based on the difference of normalized Hankel matrices between reference and damaged states, which can be easily computed. The statistical properties of the residual are reported and used for efficient hypothesis testing. Its robustness to excitation changes is shown. The proposed scheme is evaluated in numerical simulations, validating its robustness, and tested on real data sets from a full scale bridge.

Keywords: damage detection, hypothesis testing, robust residual, Hankel matrix normalization

1. INTRODUCTION

Monitoring of the structural integrity based on measurements refers to detecting changes with some damage-sensitive features derived from the data. In that context, many works employ modal parameter estimates e.g. natural frequencies [14], which can be evaluated for damages using control charts [10]. The evaluation of data-based residuals in context of SHM often relies on an outlier analysis [16], cointegration [4] and many other techniques from the statistical signal processing field.

The performance of modal parameter-based methods depends on several factors e.g. the quality of parameter estimation from the data and the ability to track the selected estimates after identifying them in the healthy state of the system. This, and the subsequent necessary automation, has been shown to be a complex task. Hence it is desirable to compute relevant features directly on the data without any need of modal parameter estimation. Together with a statistical evaluation of such features, it can yield automated damage detection. In addition to the statistical variability due to limited data length, those data-based damage features are inherently dependent on the natural changes in the ambient excitation conditions. That poses a major challenge in the evaluation of such indicators, since excitation conditions are in principle unknown and unmeasured, hence any related change may be incorrectly classified as damage. A solution to this problem lies in the design of a damage detection residual, whose mean value is independent of changes in excitation conditions.

Subspace-based damage detection methods [2, 7] are a well-known group of methods that have been successfully applied to vibration-based SHM of engineering structures, e.g. in [5]. In its classical form the subspace-based residual is linked to changes in the subspace spanned by the Hankel matrix built from the output covariance sequences of the tested data by confronting it with its left null space from the reference, undamaged, state. It is well-known that the Hankel matrices of the output covariance sequences contain information about the system matrices, which define the dynamic behavior of a structural system. However, they are dependent on the excitation conditions, which can rapidly change based on the environment. A damage detection scheme based on Hankel matrices should hence be designed to monitor the changes only in the structural system, which requires appreciation of this environmental variation.

In this context, a new residual is proposed using the difference between the Hankel matrix in the reference state and the excitation normalized Hankel matrix in the tested, potentially damaged, state. As such, the aforementioned residual is evaluated in the local asymptotic approach framework for Gaussian residuals [3]. To decide about the health of the system, the value of the resulting hypothesis test is compared to a threshold. The robustness of the new approach is achieved via a normalization scheme that is adapted from the multi-setup subspace-based system identification [6]. Subsequently, the proposed approach is tested on numerical simulations and experimental data from a full scale bridge.

2. BACKGROUND AND PROBLEM STATEMENT

In this section some background on the computation of Hankel matrices of output covariance sequences from the data are recalled. Next some numerical experiment is set up to illustrate the effect of a changing excitation properties on the empirical damage detection metric employing Hankel matrices. It is illustrated that the empirical methods do not work satisfactory when the covariance of the excitation is varying, thus this simple example should be an illustration of the new scheme proposed in the next section.

Let $R_i = \mathrm{E}(y_k y_{k-i}^{\mathrm{pc}T})$ be the theoretical covariance of the discrete measurements y_k at time lag i, also called *output covariances*, where superscript 'pc' denotes r_0 selected channels, also called projection channels. The collection of R_i can be stacked to form a block Hankel matrix $\mathcal{H} \in \mathbb{R}^{(p+1)r \times qr_0}$

$$\mathcal{H} = \begin{bmatrix} R_1 & R_2 & \vdots & R_q \\ R_2 & R_3 & \vdots & R_{q+1} \\ \vdots & \vdots & \vdots & \vdots \\ R_{p+1} & R_{p+2} & \vdots & R_{p+q} \end{bmatrix} \tag{1}$$

where p, q are parameters with usually $p = q+1$, r labels the total number of channels and r_0 denotes the number of reference channels. The estimates $\hat{R}_i$ and consequently $\hat{\mathcal{H}}$ can be computed from the sample covariance from past $\mathcal{Y}^+$ and future $\mathcal{Y}^-$ data horizons such that $\hat{\mathcal{H}} = \mathcal{Y}^+\mathcal{Y}^{-T}$ and

$$\mathcal{Y}^+ = \frac{1}{\sqrt{N}} \begin{bmatrix} y_{q+1} & y_{q+2} & \vdots & y_{N+q} \\ y_{q+2} & y_{q+3} & \vdots & y_{N+q+1} \\ \vdots & \vdots & \vdots & \vdots \\ y_{p+q+1} & y_{p+q+2} & \vdots & y_{p+q+N} \end{bmatrix}, \; \mathcal{Y}^- = \frac{1}{\sqrt{N}} \begin{bmatrix} y_q^{\text{pc}} & y_{q+1}^{\text{pc}} & \vdots & y_{N+q-1}^{\text{pc}} \\ y_{q-1}^{\text{pc}} & y_q^{\text{pc}} & \vdots & y_{N+q-2}^{\text{pc}} \\ \vdots & \vdots & \vdots & \vdots \\ y_1^{\text{pc}} & y_2^{\text{pc}} & \vdots & y_N^{\text{pc}} \end{bmatrix},$$

where $N+p+q$ is total a number of samples.

The numerical example is a theoretical 6 DOF chain-like system that, for any consistent set of units, is modeled with a proportional damping matrix, spring stiffness $k_1 = k_3 = k_5 = 100$ and $k_2 = k_4 = k_6 = 200$ and mass $m_i = 1/20$. The system is subjected to noise with a changing excitation covariance Q acting at all DOFs. Responses are measured at 1, 3 and 6 DOF. The simulation campaign consists of the reference built with $N_0 = 2 \cdot 10^6$ data points and the tested states, each simulated with $N = 10^5$ data points. Different excitation covariance scenarios are considered, namely

- first, the most simple theoretical case, when Q is the identity matrix $\mathcal{I}_6$,
- second, where every excitation amplitude is scaled with a random scalar constant a such that $Q = a \cdot \mathcal{I}_6$,
- third, the most general case, where $Q = a \cdot bb^T$ where $b \in \mathbb{R}^{6\times 6}$ is a randomly generated matrix whose entries follow a standard Gaussian distribution.

Each data set simulated beside the reference one corresponds to a different damage level occurring during the respective excitation scenario. The damage is modeled as a gradual stiffness reduction of the second (unmeasured) spring. Gaussian white noise with standard deviation equal to 5% of the standard deviation of the output is added to the response at each measurement channel.

An empirical way to counter non-complex changes in the excitation covariance is to normalize the data with e.g. the standard deviations related to the output covariance R_0, as proposed in [9]. As such, a residual for damage detection that incorporates such normalization can be developed based on the Mahalanobis distance [12] computed on estimates of the Hankel matrices from respectively reference ($\hat{\mathcal{H}}_{ref}$) and tested states ($\hat{\mathcal{H}}_{test}$). Such distance metric d writes as

$$d(\hat{\mathcal{H}}_{ref}, \hat{\mathcal{H}}_{est}) = \sqrt{\text{vec}(\hat{\mathcal{H}}_{ref}^W - \hat{\mathcal{H}}_{test}^W)^T \Sigma_{\hat{\mathcal{H}}_{ref}^W}^{-1} \text{vec}(\hat{\mathcal{H}}_{ref}^W - \hat{\mathcal{H}}_{test}^W)}\,, \tag{2}$$

where $\hat{\mathcal{H}}^W = W^{-1}\hat{\mathcal{H}}W_{\text{pc}}^{-1}$ and $\Sigma_{\hat{\mathcal{H}}_{ref}^W}$ is the covariance of the normalized Hankel matrix in the reference state, computed using the sample mean scheme e.g. after [7]. The normalization factors W and W_{pc} write as

$$W = \mathcal{I}_{(p+1)r} \otimes W_r,\; W_r = \left(\text{E}(y_k y_k^T)\right)^{1/2}$$

and

$$W_{\text{pc}} = \mathcal{I}_{(p+1)r} \otimes W_{r,\text{pc}},\; W_{r,\text{pc}} = \left(\text{E}(y_k^{\text{pc}} y_k^{\text{pc}T})\right)^{1/2},$$

where $(\cdot)^{1/2}$ denotes the matrix square root. The results from that scheme are depicted in Figure 1.

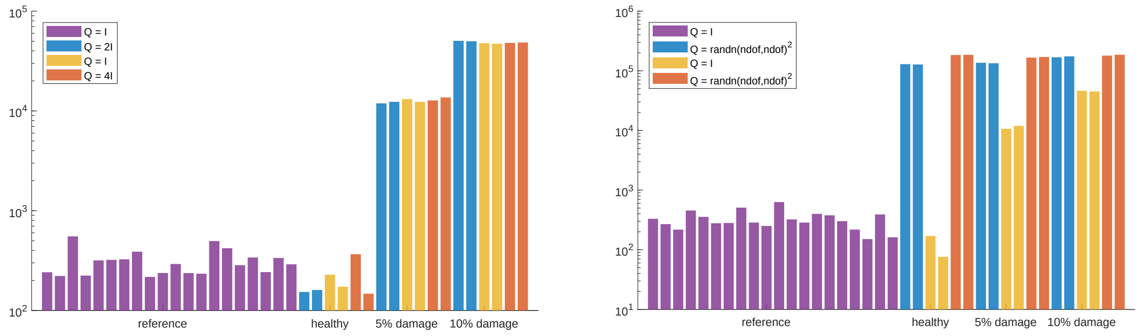

Figure 1: Damage detection with metric d (2).

When normalizing in this way, a different scaling of the noise covariance can be handled as illustrated in Figure 1 (left), whereas more complex Q matrices yield to false alarms and bad performance for the detection in Figure 1 (right). To mitigate these issues, a more robust normalization needs to be considered.

3. DAMAGE DETECTION RESIDUAL BASED ON ROBUST NORMALIZATION

In this section a new damage detection residual is introduced based on a robust normalization of the Hankel matrices. The newly developed metric fits in a well-known framework, namely the local asymptotic approach for change detection [3, 7]. Decision about the damage is taken with a statistical test which follows a χ^2 distribution, and whose mean is robust to changes in the excitation covariance under the null hypothesis. The proposed scheme is validated with a numerical simulation and tested on a full scale bridge example.

3.1. Normalization scheme

Let $\mathcal{H}_{ref}$ and $\mathcal{H}_{test}$ be the exact Hankel matrices of rank n, computed for a system subjected to excitation with different covariances Q_{ref} and Q_{test}. The SVD of the juxtaposed $\mathcal{H}_{ref}$ and $\mathcal{H}_{test}$ writes

$$\begin{bmatrix}\mathcal{H}_{ref} & \mathcal{H}_{test}\end{bmatrix} = \begin{bmatrix}U_s & U_{ker}\end{bmatrix} \begin{bmatrix} D_s & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{s,ref}^T & V_{s,test}^T \\ V_{ker,ref}^T & V_{ker,test}^T \end{bmatrix} , \tag{3}$$

where $\mathrm{rank}(\begin{bmatrix}\mathcal{H}_{ref} & \mathcal{H}_{test}\end{bmatrix}) = n$, $U_s \in \mathbb{R}^{(p+1)r \times n}$ are the left singular vectors, $D_s \in \mathbb{R}^{n \times n}$ are the non-zero singular values and $V_s^T = \begin{bmatrix}V_{s,ref}^T & V_{s,test}^T\end{bmatrix} \in \mathbb{R}^{n \times 2qr_0}$ are the right singular vectors corresponding to $\mathcal{H}_{ref}$ and $\mathcal{H}_{test}$ respectively. Now define $\mathcal{Z}_{ref} = D_s V_{s,ref}^T$ and $\mathcal{Z}_{test} = D_s V_{s,test}^T$, where both $\mathcal{Z}_{ref}$ and $\mathcal{Z}_{test}$ are full row rank. Since the converged Hankel matrices share the same observability matrix in the reference state (independently of the excitation), it holds

$$\begin{bmatrix}\mathcal{H}_{ref} & \mathcal{H}_{test}\end{bmatrix} = U_s \begin{bmatrix}\mathcal{Z}_{ref} & \mathcal{Z}_{test}\end{bmatrix} . \tag{4}$$

To compare $\mathcal{H}_{ref}$ with $\mathcal{H}_{test}$ a proper normalization of the latter matrix must be deployed, to take into account a possible change in the excitation. In order to do so, a normalization scheme is proposed that is based on the multi-setup normalization strategy from [6]. The scheme writes

$$\overline{\mathcal{H}}_{test} = \mathcal{H}_{test} \mathcal{Z}_{test}^{\dagger} \mathcal{Z}_{ref} , \tag{5}$$

where $\overline{\mathcal{H}}_{test}$ is a Hankel matrix sharing the same stochastic controlability matrix as $\mathcal{H}_{ref}$. Based on the latter expression a residual for damage detection robust to changes in the excitation covariance can be defined.

3.2. Residual definition and its asymptotic properties

The damage detection problem can be focused on a subset of parameters that are functions of the structural properties of the considered system. Typical parameters in engineering applications are the estimates of modal parameters or stiffness/mass properties of the system [1]. For that, let $\theta \in \Theta$ denote a general parameterization of the structural properties of the considered problem in a parameter space Θ. Also, let $\theta^* \in \Theta$ denote those properties under the reference state of the structure. The theoretical residual is defined as

$$\zeta(\theta) = \text{vec}\left(\mathcal{H}_{test}(\theta)\mathcal{Z}_{test}^{\dagger}\mathcal{Z}_{ref} - \mathcal{H}_{ref}(\theta_*)\right), \tag{6}$$

where $\mathcal{Z}_{ref}$ and $\mathcal{Z}_{test}$ are obtained from the SVD in (3) by truncating it at model order n. An estimate of the residual $\hat{\zeta}^{\theta}$ can be obtained from the estimates of the Hankel matrices $\hat{\mathcal{H}}_{ref}^{\theta^*}$ and $\hat{\mathcal{H}}_{test}^{\theta}$ computed from the measurement data. Here, we assume that Hankel matrices for both reference and tested states are computed on data sets with equal length N. Consequently, the system residual writes as

$$\hat{\zeta}^{\theta} = \sqrt{N}\text{vec}\left(\hat{\mathcal{H}}_{test}^{\theta}\hat{\mathcal{Z}}_{test}^{\dagger}\hat{\mathcal{Z}}_{ref} - \hat{\mathcal{H}}_{ref}^{\theta^*}\right) , \tag{7}$$

where $\hat{\mathcal{Z}}_{ref}$ and $\hat{\mathcal{Z}}_{test}$ are derived from the relation

$$\begin{bmatrix}\hat{\mathcal{H}}_{ref}^{\theta_*} & \hat{\mathcal{H}}_{test}^{\theta}\end{bmatrix} \approx \hat{U}_s \begin{bmatrix}\hat{\mathcal{Z}}_{ref} & \hat{\mathcal{Z}}_{test}\end{bmatrix} , \tag{8}$$

truncating the SVD analogous to (3) at order n. The asymptotic properties of the residual in (7) are functions of the statistical properties of the computed Hankel matrices. To recall

$$\begin{aligned} &\text{under } \mathrm{H}_0 : \sqrt{N}\text{vec}\left(\hat{\mathcal{H}}_{ref}^{\theta_*} - \mathcal{H}_{ref}(\theta_*)\right) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\mathcal{H}_{ref}(\theta_*)}) , \\ &\text{under } \mathrm{H}_1 : \sqrt{N}\text{vec}\left(\hat{\mathcal{H}}_{test}^{\theta} - \mathcal{H}_{test}(\theta_*)\right) \xrightarrow{\mathcal{L}} \mathcal{N}(\delta_{\mathcal{H}}, \Sigma_{\mathcal{H}_{test}(\theta_*)}) , \end{aligned} \tag{9}$$

where $\Sigma_{\mathcal{H}_{ref}(\theta_*)}$ and $\Sigma_{\mathcal{H}_{test}(\theta_*)}$ are the respective asymptotic covariances of $\hat{\mathcal{H}}_{ref}^{\theta^*}$ and $\hat{\mathcal{H}}_{test}^{\theta}$, and $\delta_{\mathcal{H}}$ is the mean value of the vectorized Hankel matrix in the possibly damaged state.

Lemma 1 *The residual in* (7) *is asymptotically Gaussian with the following properties*

$$\text{under } \mathrm{H}_0 : \hat{\zeta}^{\theta_*} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\zeta}) , \tag{10}$$

$$\text{under } \mathrm{H}_1 : \hat{\zeta}^{\theta} \xrightarrow{\mathcal{L}} \mathcal{N}(\delta, \Sigma_{\zeta}) , \tag{11}$$

where δ is the mean value of the residual under under H_1 *and Σ_{ζ} is its asymptotic covariance, which depends on the variance of both $\hat{\mathcal{H}}_{test}^{\theta}$ and $\hat{\mathcal{H}}_{ref}^{\theta^*}$.*

The derivation of the expressions for Σ_{ζ} is excluded from this paper due to space limitations.

3.3. Hypothesis testing

Based on the local approach, the residual in (10) is asymptotically Gaussian under both hypotheses. The decision about the damage can be achieved by applying the Generalized Likelihood ratio (GLR) test [2], which is asymptotically χ^2 distributed [8]. It writes in its simplest form as

$$t_{\text{global}}^{\text{np}} = (\hat{\zeta}^{\theta})^T \,\widehat{\Sigma}^{-1}\, \hat{\zeta}^{\theta} , \tag{12}$$

where $\widehat{\Sigma}$ is a consistent estimate of Σ_{ζ}. The performance of the proposed test is investigated on the numerical example from Section 2..

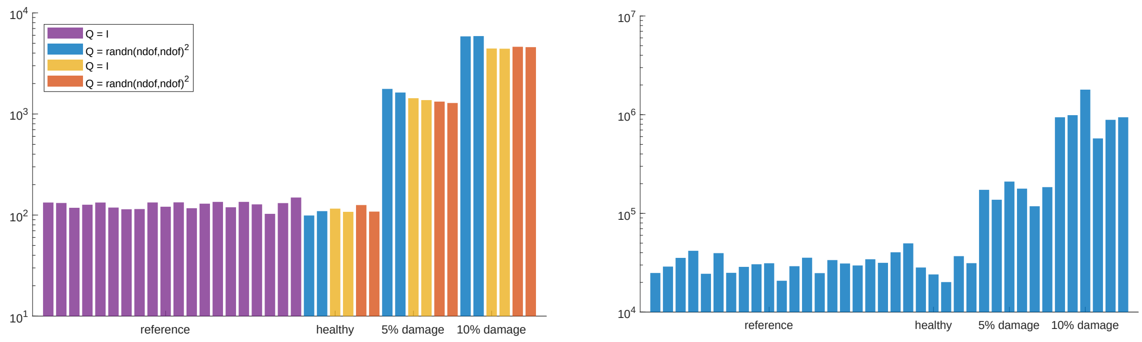

Figure 2: Damage detection based on the robust normalization. Three randomly chosen excitation levels in the tested data sets (left). Random excitation in every data set (right).

It can be observed that the test is able to separate the different damages despite being computed for different values of the excitation. Figure 2 (right) displays the same type of information for Q changing all the time for each data set. This shows that the damage can be detected without any prior knowledge of the excitation properties.

4. APPLICATION

The Z24 bridge is a benchmark for many studies involving e.g. damage detection [15] or removal of the environmental characteristics from the parameters estimated from the data [13]. Before its demolition in 1998, a progressive damage campaign was carried out and consisted of a series of ambient and forced vibration tests conducted while inducing different kinds of damage on the bridge. A complete description of this experimental campaign can be found in [11]. The progressive damage tests took place between August and September where some significant changes in the temperature conditions were experienced during its execution. The approach proposed in this paper doesn't account for the temperature variation. Therefore, only several data sets from the beginning of the experimental campaign are analyzed and assumed not to be significantly perturbed by these temperature changes.

The analyzed measurement corresponds to the mix of forced excitation from two shakers and ambient excitation from wind and traffic under the bridge. The vibration tests were conducted with 28 moving and 5 fixed sensors measuring vertical, transverse and lateral accelerations of the bridge. For the purpose of this study only the measurements from 5 fixed sensors are analyzed. The data was sampled with a frequency of 100 Hz and each measurement lasted for 655.36 seconds. A total number of 54 data sets were analyzed, from which the first 18 measurements are under healthy conditions. The first 6 data sets are selected for the reference state computation. Data sets between numbers 19 and 36 correspond to damage state 1, namely lowering one of the bridge piers by 20mm, and data sets from 37 to 54 correspond to the damage state 2, lowering the same pier by another 20mm. The view on the bridge with positions and directions of the sensors used for this study is shown on Figure 3.

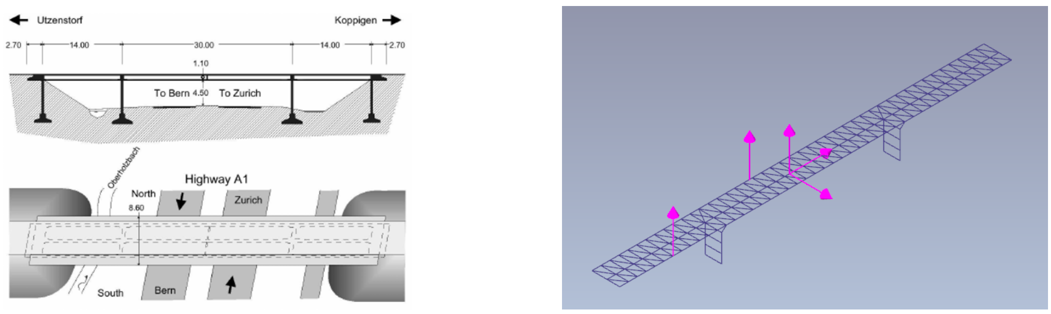

Figure 3: Front view and top views of the Z24 bridge (left). Geometry and analyzed sensors of the Z24 bridge ARTEMIS MODAL PRO 6.0 (right).

The test from (12) is computed with 10 time lags to create a Hankel matrix of model order of 6. The reference state is computed from the first 6 measurements in the healthy state. The number of blocks for the covariance of the reference and the tested Hankel matrices is selected to 400. The results are depicted in Figure 4.

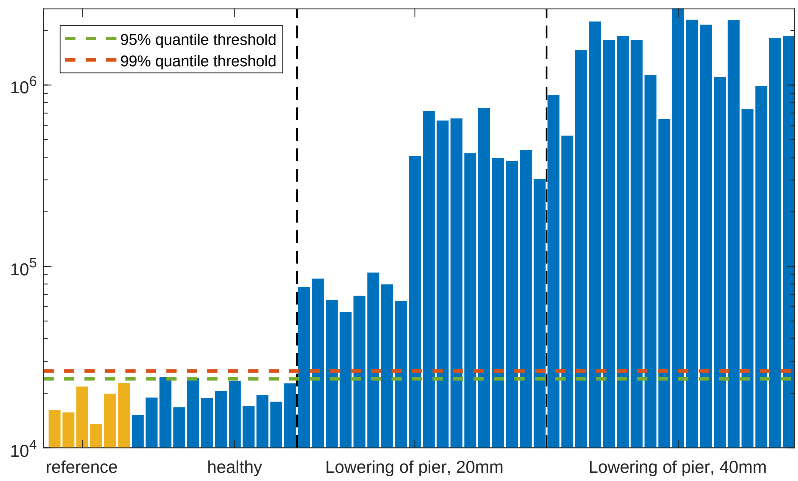

Figure 4: Damage detection in the Z24 bridge. Damage detection test after (12) .

As expected, the results exhibit no false alarms in the healthy state of the structure. Moreover, two damage levels inflicted on the bridge are clearly detected. The plotted detection thresholds correspond to two quantiles, 95% and 99%, of the empirical distribution of the test values in the reference state. Two different levels of the χ^2 test for the damage state 1 might indicate the lowering of the bridge pier in two steps by 10mm.

5. CONCLUSIONS

In this paper we proposed a new damage detection residual based on the difference of Hankel matrices in different states of the structure. The robustness of the new approach is achieved via a robust normalization scheme. It is empirically shown that the mean value of the proposed test under healthy conditions of the system does not depend on the variances of the excitation. The new method has been evaluated on numerical simulations of a simple system and on full scale experimental data from the Z24 bridge. Both cases represent case studies under complex excitation conditions. The experimental results exhibit no outliers in the healthy state, which helps to avoid false alarms and to define an accurate detection threshold.

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